Uncertain altruism and the provision of long term care*

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Abstract

This paper studies the role of private and public long term insurance programs in a world in which family assistance is uncertain. Benefits are paid in case of disability but cannot be conditioned (directly), due to moral hazard problems, on family aid. Under a topping up scheme, when the probability of altruism is high, there is no need for insurance. At lower probabilities, insurance is required, though not full insurance. This can be provided either privately or publicly if insurance premiums are fair, and publicly otherwise. Moreover, the amount of LTC insurance varies negatively with the probability of altruism. With an opting out scheme, if the degree of altruism is high, the optimal policy is to provide parents with full LTC insurance with altruistic children helping their parents. Otherwise, there can be two types of equilibrium. In one, there is full LTC insurance but with no assistance from children. In the other, the LTC insurance is less than full and just low enough to induce altruistic children to help. When the equilibrium entails assistance from altruistic children, no private insurance is used even if private insurance markets are fair. If the equilibrium is one without aid from children, public insurance no longer dominates private insurance if the latter is fair.

JEL classification: H2, H5.

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1 Introduction

Long-term care (LTC) is the provision of assistance and services to people who, because of disabling illnesses or conditions, have limited ability to perform basic daily activities such as bathing and preparing meals. It is a problem mainly, though not exclusively, for the elderly. As people have come to live longer, the demand for LTC services by the elderly population has grown substantially in recent years—a trend likely to further increase and intensify in future. There are two related reasons for this. First, LTC needs start to rise exponentially from around the age of 80 years old; second, the number of persons aged 80 years and above are growing faster than any other segment of the population. As a consequence, the number of dependent elderly is expected to more than double by 2050 in most countries. This will exacerbate the current pressures on the demand for LTC services and lead to new challenges for these countries and their governments.

There are, currently, three institutions that finance and provide LTC services: the family, the market, and the state. The majority of the dependent population receiving long-term care at home rely exclusively on assistance from family members, mainly women; this is often referred to as "informal care". This avenue for LTC provision is, unfortunately, facing a number of formidable challenges: drastic changes in family values, increasing number of childless households, mobility of children, and growing rate of market activity on the part of women (particularly those aged 50–65). As a consequence, the number of dependent elderly who will not be able to count on the assistance of family members is likely to increase. This creates a pressing demand on the other two institutions, the market and the state, to offer either a substitute or a complement to what the family has thus far been providing by way of long term care.

The aim of this paper is to highlight and study the challenge posed by the idea that family solidarity is uncertain. There are multifold reasons for reasons for this. First, there are pure demographic factors such as widowhood, the absence or the loss...
of children. Divorce and migration can also be put in that category. Other causes are conflicts within the family or financial problems incurred by children, which prevent them from helping their parents. Whatever the reason, the possibility of solidarity default requires people to take appropriate steps such as purchasing private insurance, self-insuring or relying on some public insurance or assistance scheme. What makes the problem particularly daunting and ripe for government intervention is the fact that there exists no good insurance mechanism to protect individuals against all sorts of default of family altruism.

The uncertain nature of family solidarity has interesting implications for a number of public policy questions. Chabbakatri et al. (1993) have studied the consequences of uncertain altruism for old age retirement. They show that this leads individuals to increase their retirement saving. In the same vein, Leroux and Pestieau (2011) show that such uncertainty induces individuals to vote for a social security scheme. Yet, in developed countries, family solidarity is no longer a mechanism for retirement support. That it remains the main channel for assistance in case of dependency, calls for a serious study of its implications in this context.

We study the role of private and public insurance programs in a world in which family assistance is uncertain. We do this by modeling the behavior and welfare of one single generation of “parents” over their life cycle. When they are young, they work, consume, and save for their retirement. In retirement, they face a probability of becoming dependent. This probability is exogenously determined and parents cannot affect it through their behavior (when young or when old). If they become dependent, parents face yet another uncertainty. They may or may not receive assistance from their children. Many factors affect the children’s behavior. Some causes of altruism default are purely exogenous, but others can be influenced by the parents. Investment in the children’s education and inculcating values in them through one’s own behavior
are such mechanisms. In this paper, we study the problem assuming an exogenously given probability for the default of altruism on the part of the children. We shall leave the setting where parents can affect this probability to another study.

An important feature of our study is that we do not rule out private insurance markets by fiat. Indeed, we allow for the possibility of parents insuring themselves against becoming dependent. Plainly, however, moral hazard problems preclude the development of insurance markets against the default of altruism per se as opposed to dependency. For the same reasons, the government cannot condition its assistance to the old on the default of altruism; only on age-old dependency. Within this framework, we provide answers to two broadly defined questions.

One question is the general need for insurance and how it should be provided: privately or publicly. We study the conditions under which private savings will or will not be enough for the three states of the world parents face in retirement (autonomy and dependency with or without assistance from children). In the latter case, we examine when parents can rely on private insurance markets to secure the extra resources they want in case of dependency (because of the possibility of altruism default on the part of the children). We also discuss the circumstances that call for the government to step in and provide the needed assistance.

The second broad question we address concerns the nature of public assistance. One possibility is for the government to provide all dependent parents with monetary help while allowing them to top this up as they see fit. Another possibility is for the government to provide every dependent parent who asks for it a "minimal" care facility. If this is deemed insufficient the parent will have to opt out and use his own resources, and their children's, to purchase whatever home care services he needs (without any help from the government). The dependent parent consumes either one or the other. We examine and compare the properties of these two schemes.

On this, see on this Kotlikoff and Spivak (1981) or Cox and Stark (2005).
In searching a role for the government, we confine ourselves to scenarios wherein the cost of financing the LTC program is borne by the potential beneficiaries themselves (and not by their children or future generations). In this way, one can zero-in on the insurance reasons for such programs rather than compounding this with issues that arise from wealth transfers across generations. Consequently, we model the behavior and welfare of one single generation of parents over their life cycle. Our interest in their children is limited only to their role in providing assistance to their parents. As a consequence, the welfare of the grown-up children does not figure in government's objective function; only the expected utility of parents. Nor do the children pay any taxes to finance the LTC program (otherwise, they become a "costless" source of taxation to provide benefits for their parents).

In answering the first set of questions we have asked, we find that the scheme the government adopts, "topping up" or "opting out", has an important bearing on the question of who should provide LTC; the market or the state. Specifically, under the topping up scheme, if the probability of altruism is high there is no need for insurance regardless of who provides it. All assistance is provided through one's children and private savings. At lower probabilities, LTC insurance is called for; albeit one that is less than full. Moreover, the amount of insurance varies negatively with the probability of altruism. If private insurance markets for dependency are fair, private insurance will suffice (although public assistance is just as good). At higher than fair insurance premiums, on the other hand, public assistance dominates private insurance.

With an opting out scheme, different types of equilibrium emerge. All require LTC insurance regardless of the probability of altruism (in contrast to the topping up scheme). If altruistic children do not consider full LTC insurance to be enough, the optimal policy is to provide parents with full LTC insurance and there will be an equilibrium wherein altruistic children do help their parents. In this equilibrium, the amount of LTC insurance and the expected utility of parents are higher under the opting out scheme.
as compared to the topping up scheme. Private savings, on the other, will be smaller (regardless of the probability of altruism).

If altruistic children do not help their parents when they are given full LTC insurance by the government, there can be two types of equilibrium. One calls for full LTC insurance (but with no assistance from their children). In the other equilibrium, the government restricts the LTC insurance to less than full and just low enough to induce altruistic children to help. Both equilibria are possible depending on the parameterization of the model.

Most interestingly, we find that when the equilibrium entails assistance from altruistic children, no private insurance is used even if private insurance markets are fair. All assistance should be provided publicly. On the other hand, if the equilibrium is one without assistance from altruistic children, public insurance no longer dominates private insurance if the latter is fair.

2 The model

Consider a setup wherein (i) in period 0, the government formulates and announces its tax/transfer policy; (ii) in period 1, a young working parent and a child appear on the scene and the parent decides on his present and future consumption levels; in period 2, the parent has grown old and is retired, and the child who has turned into a working adult decides if he wants to help his parents. We will not be concerned with what will happen to the grown-up child when he turns old, nor with any future generations. Two uncertain events give rise to the problem that we study. One concerns the health of the parent in old age. He may be either “dependent” or “independent”. Denote the probability of dependency by \( \pi \); naturally, the parent will be independent with probability \( 1 - \pi \). We assume that \( \pi \) is exogenously given. The second source of uncertainty concerns the help that the parent might get from his children if dependent. Denote the probability that a child is altruistic towards his parents by \( p \) and the probability that
he is not by $1 - p$. We shall assume that $p$ is also exogenously fixed.\footnote{In a companion paper, Cremer \textit{et al.} (2012), we will drop this assumption and allow parents to affect this probability by spending time with their children when they are growing up. There we also consider the case where parents are heterogeneous in their earning abilities.}

Parents have preferences over consumption when young, $c$, and consumption when old, $d$; there is no disutility associated with working. We assume, for simplicity, that the parents' preferences are quasilinear in consumption when young. With $p$ assumed fixed, labor supply is also fixed as there is no other usage for a parent's time.

Government's policy consists of levying a tax at rate of $\tau$ on the parents' wage, $w$, to finance public provision of dependency assistance, $g$. We shall refer to $g$ as the LTC insurance. As far as private insurance is concerned, it is easier to rule it out initially for modeling purposes. Having established the need for insurance, we examine if it can be decentralized through private insurance markets.

Denote the level of assistance an altruistic child would give his parents by $a$, savings by $s$, and set the rate of interest on savings at zero. One can then represent the parent's preferences by means of the expected utility,

$$EU = w (1 - \tau) T - s + (1 - \pi)U(s) + \pi [pH(s + g + a) + (1 - p)H(s + g)].$$

(1)

We shall assume that the grown-up children too have quasilinear preferences represented by

$$u = y - a + \beta H(s + g + a),$$

(2)

where $y$ denotes their income with $\beta$ being the degree of their altruism towards their parents. Those who are not altruistic towards their parents have a $\beta = 0$.

To determine the government's policy we start by studying the last stage of our decision making process. This is when the grown-up children decide on the extent of their help to their parents, if any.
2.1 Stage 3: The child’s choice

The altruistic child allocates an amount $a$ of his income $y$ to assist his dependent parent (given the parent’s savings $s$ and the government’s provision of $g$). Its optimal level, $a^*$, is found through the maximization of equation (2). The first-order condition with respect to $a$ is, assuming an interior solution,3

$$-1 + \beta H'(s + g + a) = 0.$$ 

It follows from this condition that $a^*$ satisfies

$$s + g + a^* = (H')^{-1} \left( \frac{1}{\beta} \right) \equiv m(\beta). \quad (3)$$

Observe also that with $H''(\cdot) < 0$,4

$$\frac{\partial m}{\partial \beta} > 0.$$

2.2 Stage 2: The parent’s choice

Recall that the parent may experience two states of nature when retired: dependency with probability $\pi$ and autonomy with probability $(1 - \pi)$. Substituting for $a^*$ from (3) in the parent’s expected utility function (1), we have

$$EU = w (1 - \pi) T - s + (1 - \pi) U(s) + \pi [pH(m(\beta)) + (1 - p) H(s + g)]. \quad (4)$$

3 A sufficient condition for an interior solution is to have

$$\left. \frac{du}{da} \right|_{a=0} = -1 + \beta H'(s + g) > 0,$$

or

$$H'(s + g) > \frac{1}{\beta}.$$

4 This follows from differentiation of the first-order condition that results in

$$H''(s + g + a^*) \frac{\partial m(\beta)}{\partial \beta} = -1 \frac{1}{\beta^2}.$$
Maximizing $EU$ with respect to $s$, and assuming an interior solution, the optimal value of $s$ satisfies

$$(1 - \pi) U'(s) + \pi (1 - p) H'(s + g) = 1. \quad (5)$$

Denote the solution to equation (5) by $s(p; g)$. Substituting $s(p; g)$ for $s$ in (5), the resulting relationship holds for all values of $p$ and $g$. Differentiating this relationship partially with respect to $p$ and $g$ yields,

$$\frac{\partial s}{\partial p} = \frac{\pi H'(s + g)}{(1 - \pi) U''(s) + \pi (1 - p) H''(s + g)} < 0, \quad (6)$$

$$\frac{\partial s}{\partial g} = \frac{-\pi (1 - p) H''(s + g)}{(1 - \pi) U''(s) + \pi (1 - p) H''(s + g)} < 0. \quad (7)$$

Thus a parent's savings move negatively with the probability of altruism and with the government's level of assistance.

### 2.3 Stage 1: The optimal policy

The government chooses $\tau$ and $g$ to maximize $EU$, as optimized by the parents in stage 2, subject its budget constraint

$$\tau wT = \pi g. \quad (8)$$

Substituting for $\tau$ from (8) into the parents' optimized value of $EU$, the government chooses $g$ to maximize

$$L = wT - \pi g - s(p; g) + (1 - \pi) U(s(p; g)) + \pi [pH(m(\beta)) + (1 - p) H(s(p; g) + g)]. \quad (9)$$

Differentiating $L$ with respect to $g$ yields, using the envelope theorem,

$$\frac{dL}{dg} = \pi [(1 - p) H'(s(p; g) + g) - 1]. \quad (10)$$

There are two possible outcomes depending on the sign of $dL/dg$ at $g = 0$. 
One possible outcome is that

\[(1 - p) H'(s(p)) - 1 \leq 0, \quad (11)\]

where \(s(p) \equiv s(p;0)\). This condition holds in the neighborhood of \(p = 1\). Under this condition, \(g = \tau = 0\) and no LTC insurance is called for. The consumption levels provided through one’s own savings, and help from altruistic children, are sufficient (albeit, if \(p < 1\), ex post there will be some people with “insufficient” means).\(^6\)

The second outcome occurs if

\[(1 - p) H'(s(p)) - 1 > 0. \quad (12)\]

This condition is necessarily satisfied for some \(p\) as long as \(H'(s(0)) > U'(s(0))\).\(^7\) In this case, there will be an interior solution for \(g\), and \(\tau\), characterized by

\[H'(s(p;g) + g) = \frac{1}{1-p} > 1. \quad (13)\]

Consequently, there is less than full insurance.\(^8\) Substituting from (13) into (5), it is also the case that

\[U'(s(p;g)) = 1. \quad (14)\]

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\(^5\)The condition is satisfied at \(p = 1\) and by continuity in the neighborhood of \(p = 1\).

\(^6\)One explanatory reason mentioned for the LTC insurance puzzle is family solidarity (as well as the more well-known annuity puzzle); see Pestieau and Ponthière (2010). Our result suggests that even uncertain family solidarity may explain the puzzle.

\(^7\)Observe that

\[\frac{d}{dp} [(1 - p) H'(s(p)) - 1] = -H'(s(p)) + (1 - p) H''(s(p)) s'(p),\]

which is negative at \(p = 1\). Consequently, as \(p\) becomes smaller than one, \((1 - p) H'(s(p)) - 1\) initially increases so that at some point might change its sign. Condition \(H'(s(0)) > 1\) ensures that this will happen. In turn, from (5), this condition is equivalent to assuming that \(H'(s(0)) > U'(s(0))\). The need for this assumption arises because with \(H''(s(p)) < 0\) and \(s'(p) < 0\), the sign of the above derivative while initially negative is in general indeterminate so that it may not always remain negative. Even if it does, it does not guarantee that \((1 - p) H'(s(p)) - 1\) necessarily changes sign.

\(^8\)Full insurance requires the equality of \(H'(\cdot)\) with the marginal utility of current consumption which, with quasilinear preferences, is equal to one.
Interestingly too, it follows from equation (13) and (14) that the publicly-provided LTC insurance \( g \), and the tax rate \( \tau \), decrease with \( p \). Specifically, differentiating (14) with respect to \( p \) yields\(^9\)

\[
\frac{dg}{dp} = \frac{H'(s + g)}{(1 - p) H''(s + g)} < 0. \tag{15}
\]

It is important to understand what forces are behind this result. On the one hand, a higher \( p \) results in smaller private savings \( s(p; g) \). There will thus be a negative impact on the available resources for consumption in case of altruism default, \( s + g \), and an increase in the value of \( H'(s + g) \). This, in turn, induces the government to provide more \( g \) (to compensate for the reduction in \( s \)). On the other hand, providing more \( g \) for dependent parents who do not get help from their children is also accompanied by providing more \( g \) for dependent parents who do receive aid from their children. Now while parents bear all the cost of providing more \( g \) in terms of a loss in their present day consumption, the beneficiary of more \( g \) also includes the altruistic children who would have otherwise provided the extra \( g \) to their parents. These additional costs to the parents dominate and result in a negative relationship between \( g \) and \( p \).

Finally, with our findings that \( \frac{dg}{dp} < 0 \) and \( g = 0 \) at high values of \( p \), there must exist a \( p = \hat{p} \) at which the switch from one outcome to the other occurs. This is when condition (11) holds as an equality:

\[
H'(s(\hat{p})) = \frac{1}{1 - \hat{p}}. \tag{16}
\]

\(^9\)Upon differentiating (14) one gets

\[
U''(s(p; g)) \left[ \frac{\partial s}{\partial p} + \frac{\partial s}{\partial g} \frac{dg}{dp} \right] = 0.
\]

It follows from this equation that

\[
\frac{dg}{dp} = -\frac{\frac{\partial s}{\partial p}}{\frac{\partial s}{\partial g}} = -\frac{\pi H'(s + g)}{(1 - \pi)U''(s) + \pi(1 - p)H''(s + g)}.
\]

Simplification results in (15).
Observe also that at this price, we have from equation (14) that

\[ U' (s (\hat{p})) = 1. \]  \hspace{1cm} (17)

### 2.4 The question of private insurance

Assume \( 0 \leq p < \hat{p} \) so that dependent parents need additional assistance beyond their own savings when assistance is not forthcoming from their children. The question is if they can secure this through private insurance markets. To answer this question, denote the amount of insurance the parent may buy against old-age dependency, if it is optimal to have private insurance markets, by \( \theta \) and its unit price by \( q \). First, observe that allowing private insurance markets does not change the consumption level of a parent receiving assistance from his children; it remains at \( m (\beta) \).\(^{10}\) Second, let \( e \) denote the government’s assistance above the privately-purchased insurance, \( \theta \), so that \( g = \theta + e \). Reconsidering the parents’ optimization problem, one can show that savings depends on \( \theta + e \) and not its division between public and private insurance.\(^{11}\)

Now consider the expected utility of the parent, if private insurance markets are operative. Fix the total amount of LTC insurance provided both privately and publicly, for the given \( p \), at its optimal level derived for when all insurance is provided by the government. That is set, \( \theta + e = g^* \). Under this circumstance, parents’ savings would

\(^{10}\)Let \( e \) denote the government’s assistance above the privately-purchased insurance, \( \theta \). The children’s optimization level results in the first-order condition \(-1 + \beta H' (s + \theta + e + a) = 0\), so that the total consumption of the parent, \( s + \theta + e + a \), does not change.

\(^{11}\)Expected utility of the parent is now written as

\[ EU = w (1 - \tau) T - q\theta - s + (1 - \pi) U (s) + \pi [p H (m (\beta)) + (1 - p) H (s + \theta + e)]. \]

This leads to the following first-order condition with respect \( s \),

\[ (1 - \pi) U' (s) + \pi (1 - p) H' (s + \theta + e) = 1. \]

Comparing this equation with (5) indicates this point.
continue to be what it was when all LTC insurance is provided publicly. We thus have

\[ EU = w (1 - \tau) T - q^* \theta - s^* + (1 - \pi) U (s^*) + \pi [pH (m(\beta)) + (1 - p) H (s^* + g^*)], \]

(18)

where a "star" on top of a variable indicates its fixed optimal value. With the parents' purchasing \( \theta \), the government must finance only \( e = g^* - \theta \) in assistance to dependent parents. This changes the government's budget constraint to

\[ \tau w T = \pi (g^* - \theta). \]

(19)

Substitute for \( \tau \) from (19) into (18). Upon simplification, we have

\[ EU = w T - \pi g^* + (\pi - q) \theta - s^* + (1 - \pi) U (s^*) + \pi [pH (m(\beta)) + (1 - p) H (s^* + g^*)]. \]

(20)

At \( \theta = 0 \), we have the optimal solution with insurance being provided solely by the government. The question is if having \( \theta > 0 \) benefits or harms the parents. To answer this question, differentiate (20) with respect to \( \theta \). This yields

\[ \frac{\partial EU}{\partial \theta} = \pi - q. \]

(21)

The result in (21) makes perfect sense. Both the private sector and the government provide insurance against dependency and not lack of altruism. The per unit cost through the private sector is \( q \) and through the government, implicitly, \( \pi \). If the private insurance price is "fair" then \( q = \pi \) and one is just as good as another. Under this circumstance, private markets suffice and there is no additional need for government intervention. On the other hand, if \( q > \pi \), providing assistance through private markets can only hurt the parents. Insurance should be provided only publicly.

The main results of this section are summarized in the following proposition.
Proposition 1 Consider a topping up scheme.

Let \( s(p) \) solve
\[
(1 - \pi) U'(s(p)) + \pi (1 - p) H'(s(p)) = 1.
\]

Define \( \hat{p} \) as the solution to
\[
H'(s(p)) = \frac{1}{1 - p}.
\]

(i) If
\[
\hat{p} \leq p \leq 1,
\]
there is no need for insurance regardless of who provides it. All assistance is provided through one’s children and private savings. In other words, even uncertain family solidarity may explain the LTC insurance puzzle.

(ii) If
\[
0 \leq p < \hat{p},
\]
there will be need for LTC insurance.

(a) If \( q = \pi \), i.e., if private insurance markets for dependency are fair, private insurance will suffice (although public assistance is just as good). On the other hand, if \( q > \pi \), public assistance dominates private insurance.

(b) The optimal amount of LTC insurance \( g \) is characterized by
\[
H'(s(p; g) + g) = \frac{1}{1 - p} > 1,
\]
and there is less than full insurance.

(c) The amount of LTC insurance \( g \) varies negatively with the probability of receiving aid from one’s children, \( p \).

3 Opting out

We now assume that, if made dependent, parents will need a particular good/service, \( x \). Specifically, one can think of purchasing home care services using one’s own, and
one's children's, resources versus government provision of a minimum facility. The crucial point is that one consumes either one or the other. One cannot top up what the government provides.

Denote government’s provision by $z$. When providing such a facility, the government taxes away the recipient’s resources (savings and any private insurance that he may have purchased). Consequently, $x = s + a$ if LTC insurance is provided by the altruistic children and $x = z$ if it is provided by the government. Clearly, the dependent parents with non-altruistic children end up with a consumption level $x = z$. What the consumption level of parents with altruistic children will be is not clear; however. That depends on the behavior of these children. Whereas under a topping up scheme an altruistic child decides on how much to supplement the government’s provision; under an opting out scheme, he would have to decide between his own assistance versus that of the government’s. Thus the parents end up either with some assistance from their altruistic children or nothing at all. As in the previous section, we start with stage 3 and the children’s decision.

3.1 Stage 3: The child’s choice

The optimal level of family assistance $a^*$, if there is to be any, is defined as the solution to the maximization of

$$u = y - a + \beta H (s + a).$$

This is given by

$$s + a^* = (H')^{-1} \left( \frac{1}{\beta} \right) \equiv m(\beta).$$

Observe that while $a^*$ is now given by $m(\beta) - s$ as opposed to $m(\beta) - (s + g)$ in the previous section, the actual consumption level of the dependent parent when he is being assisted by his children remains the same. Put differently, the child fully makes up for what the government does not provide.
Clearly, the altruistic child chooses \( a^* \) if and only if it gives him more utility than the option of no assistance. However, whereas in the topping up case \( a^* = m(\beta) - (s + g) \) moved continuously with \( g \); here \( a^* \) is independent of \( z \) and the child faces a discontinuous choice. This is settled by comparing his utility if he chooses his own assistance,

\[
u = y - a^* + \beta H(s + a^*),
\]

versus his utility if he goes with government’s assistance for his parents,

\[
u = y + \beta H(z).
\]

The two yield the same utility level if

\[
H(z) = H(s + a^*) - \frac{a^*}{\beta} = H(m(\beta)) - \frac{m(\beta) - s^*}{\beta}.
\]

Denote the solution to equation (25) by \( \hat{z} \). If the \( z \) offered by the government falls short of \( \hat{z} \), the altruistic child provides his parents with \( a^* \) in aid; if \( z \) exceeds \( \hat{z} \), the child opts for no assistance.

Observe also that if an altruistic child opts to help his parents, he must be providing them with a higher consumption level than what they can get from the government. That is, it must be the case that \( s + a^* > z \).\(^{12}\) An implication of this is that parents who receive aid from their children will never ask the government for help; the program is self-targeted.

\(^{12}\) This follows because when the altruistic child decides to help his parent, it follows from (23)–(24) that

\[
H(s + a) - H(z) > a/\beta > 0.
\]
3.2 Stage 2: The parent’s choice

Assume first that altruistic children come to the help of their parents. With \( s + a^* = m(\beta) \), the expected utility of the parent is

\[
EU = w (1 - \tau) T - s + (1 - \pi) U(s) + \pi [pH(m(\beta)) + (1 - p) H(z)].
\]

The parent's first-order condition with respect to \( s \) then results in\(^\text{13} \)

\[
U'(s) = \frac{1}{1 - \pi}.
\]

Note that, unlike the topping up case, the solution for \( s \) is independent of the probability of altruism, \( p \), and government’s level of assistance, \( z \). We denote the solution to equation (26) by \( s^* \).

Next consider the case where altruistic children do not help their parents. The parent's expected utility is now given by

\[
EU = w (1 - \tau) T - s + (1 - \pi) U(s) + \pi H(z).
\]

This leads to the first-order condition,

\[
U'(s) = \frac{1}{1 - \pi}.
\]

This is identical to the optimal condition for \( s \) when altruistic children help their parents, equation (26). We thus have the same solution, \( s^* \).

3.3 Stage 3: The optimal policy

The determination of optimal policy is a more complicated undertaking under the opting out scheme as compared to the topping up scheme. The government chooses its policy anticipating what the children and parents do. The complicating factor here is the discontinuous manner with which the government can, through its policy choice, induce

\(^{13}\)An interior solution is guaranteed by assuming that \( U'(0) > 1/(1 - \pi) \).
altruistic children to move from providing a given level of assistance to none, and vice versa. One observation helps in figuring out if the government may want to opt for an equilibrium with or without assistance from children: For a given level of $z$, parents enjoy a higher expected utility with children's assistance than without.\footnote{For the same $z$, the expected utility of parents will be higher by $\pi p \{ [H (m (\beta)) - H (z)] + (z - s^*) \} > 0$, under an equilibrium with assistance from altruistic children as compared to no assistance.} This follows because help from children is "free" but help from the government comes at the cost of taxing the parents. One thus searches for the optimal solution assuming assistance from children. If this solution is unconstrained, it is the unconditional optimal policy. If it is constrained, i.e. if the solution is to set $z = \hat{z}$, then one has to compare this solution with the best the government can do without children's assistance. Figures 1–3 depict the three possible solutions.

Figure 1: Unconstrained equilibrium with aid from altruistic children.
Figure 2: Constrained equilibrium with aid from altruistic children.

Figure 3: Equilibrium with no aid from altruistic children.
4 Optimal opting out policy: altruistic children help

With targeted assistance, and help from altruistic children, the government’s budget constraint changes to

$$\tau wT + \pi (1 - p) s = \pi (1 - p) z,$$

(27)
as only parents who are not helped by their children ask for assistance (and pay the additional taxes). The government chooses $z$ to maximize\(^ {15}\)

$$\mathcal{L} \equiv w (1 - \tau) \bar{T} - s^* + (1 - \pi) U (s^*) + \pi [pH (m(\beta)) + (1 - p) H (z)]$$

$$= wT - \pi (1 - p) z + \pi (1 - p) s^* - s^* +$$

$$(1 - \tau) U (s^*) + \pi [pH (m(\beta)) + (1 - p) H (z)],$$

where $s^*$ is the solution to (26). Given the assumption of help by children, the solution to the government’s optimization problem must satisfy $z \leq \hat{z}$ condition.

Differentiating $\mathcal{L}$ with respect to $z$ yields

$$\frac{\partial \mathcal{L}}{\partial z} = \pi (1 - p) [H' (z) - 1].$$

(28)

The crucial difference between the first-order condition (28) and its counterpart under the topping up scheme, equation (10), is that here $z$ is always greater than $s^*$ as long as $\frac{\partial \mathcal{L}}{\partial z} \big|_{\tau=0} = H' (s^*) > 1.16$ The optimality of a positive LTC insurance, $z - s$, does not rest on the value of $p$. We will come back to this point below. There are two possible solutions depending on whether or not the $z \leq \hat{z}$ condition is binding at the solution.

4.1 Non-restricted solution

If $z \leq \hat{z}$ condition is non-binding the optimal solution is characterized by

$$H' (z^*) = 1.$$

(29)

\(^{15}\)Except for $p = 1$ at which $\tau = z = 0$ and the government does not have an optimization problem.

\(^{16}\)With $U' (s^*) = 1/(1 - \pi) > 1$, this condition is necessarily satisfied as long as $H' (s^*) > U' (s^*)$.\footnote{19}
Introduce \( g \) to denote the \textit{net} contribution of the government to the dependent parent who seeks help. This is what the government provides net of the parent's savings, \( s \); it is the LTC insurance. Hence\(^{17}\)

\[
g = z - s. \tag{30}
\]

Substituting from (30) into (29), while setting \( s = s^* \), the optimal \( g \) is characterized by

\[
H'(s^* + g^*) = 1,
\]

and we have full LTC insurance.

With \( z^* \) and \( s^* \) not varying with \( p \), the optimal \( g \) is also independent of \( p \). Put differently, the probability of altruism has no bearing on the LTC insurance the government provides a dependent parent (who asks for it). This is in sharp contrast to the topping up scheme in which \( g^* \) varies negatively with \( p \). The difference arises because under opting out, the government does not have to worry that its insurance intended for the default of altruism is automatically provided to all dependent parents. With no leakage of benefits to the parents who are helped by their altruistic children, and thus indirectly the altruistic children themselves, the probability of altruism does not matter.\(^{18}\)

Interestingly, though, this particular solution is rather unrealistic. It appears unlikely to have the government provide full LTC insurance and still have the children help their parents on top of that. Such a solution implies an extremely high degree of altruism. Specifically, if altruistic children consider government provision insufficient and substitute their own help, it must be the case that \( a^* + s^* > z^* \). The concavity of \( H(\cdot) \) then implies \( H'(a^* + s^*) < H'(z^*) \). Now we have, from the first-order condition (22) for altruistic children providing help, \( H'(a^* + s^*) = 1/\beta \); we also have \( H'(z^*) = 1 \).\(^{19}\)

\(^{17}\) If there is private insurance of the amount \( \theta \), the dependent parent will have \( s + \theta + a \) if receiving assistance from his children, and \( z \) with the default of altruism. However, in this case \( \theta \) is also taxed away so that \( g = z - s - \theta \).

\(^{18}\) Except, of course, if \( p = 1 \) and no public LTC insurance is required.
Consequently, $1/\beta < 1$ so that $\beta > 1$. This means, rather unreasonably, that children care about their parents more than the parents care about themselves. One may refer to such children as "super altruistic".

4.1.1 Opting out versus topping up

Denote the optimal solutions under topping up and opting out schemes (with assistance from altruistic children), by superscripts $\text{OO}$ and $\text{TU}$. From equations (13)–(14), which characterize the optimal level of government provision is characterized by

$$
H'\left( s(p;g^{TU}) + g^{TU} \right) = \frac{1}{1-p} > 1,
$$

$$
U'\left( s(p;g^{TU}) \right) = 1.
$$

Similarly, from equations (26), (29), and (30),

$$
H'(s^{OO} + g^{OO}) = 1,
$$

$$
U'(s^{OO}) = \frac{1}{1-\pi} > 1,
$$

where $s^{OO} = s^*$. Comparing the equations for $H'(\cdot)$ under the two schemes, and the equations for $U'(\cdot)$, it follows from the concavity of $H(\cdot)$ and $U(\cdot)$ that

$$
\begin{align*}
\left( s(p;g^{TU}) + g^{TU} \right) &< s^{OO} + g^{OO}, \\
\left( s(p;g^{TU}) \right) &> s^{OO}.
\end{align*}
$$

These results are interesting. They tell us that, assuming optimal government's policy, parents who are not helped by their children have a smaller consumption level if dependent and a higher consumption level if independent, under the topping up scheme as compared to the opting out scheme.

---

19 On the other hand, one could also argue that Alzheimer patients do not care much about what happens to them but their children want them to be cared for in a dignified and decent manner. We remain agnostic about this possibility and do not a priori rule out $\beta > 1$. 21
In turn, these inequalities imply

\[ g^{TU} < g^{OO}. \]

Net assistance to dependent parents who receive no help from their children is always smaller under the topping up scheme as compared to opting out scheme. Finally, observe that one can always implement the optimal topping-up policy under opting out with some resources left over (as the children make up for what the government pays under topping up but not under opting out). Hence

\[ EU^{TU} < EU^{OO}. \]

4.2 Restricted solution

If \( z^* > \tilde{z} \), then \( z^* \) will be unavailable as long as altruistic children help their parents. Under this circumstance, the optimal value of \( z \) is \( \tilde{z} \). As was the case with \( z^* \), here too (i) \( \tilde{z} > s^* \) so that there is positive LTC insurance; (ii) the amount of insurance, \( \tilde{z} - s^* \) is independent of \( p \). The main difference between \( \tilde{z} \) and \( z^* \) is that, with \( \tilde{z} < z^* \), we will have

\[ H'(\tilde{z}) > H'(z^*) = 1. \]

The solution entails less than full insurance. Moreover, except for savings, our comparisons between topping up and opting out solutions are no longer necessarily valid.

4.3 Private insurance

Consider again the expected utility of the parent, if private insurance markets are operative, and fix the level of \( z \) at its optimal level. Denote this by \( Z \) where \( Z \) stands for \( z^* \) or \( \tilde{z} \) depending on whether we have a non-restricted or restricted optimum. As previously, the consumption level of a parent receiving assistance from his children remains the same at \( m(\beta) \). Parents' savings will also remain at \( s^* \). We have

\[ EU = w (1 - \tau) T - q\theta - s^* + (1 - \pi) U(s^*) + \pi [pH(m(\beta)) + (1 - p) H(Z)]. \]
However, if parents buy $\theta$ in private insurance, given that the government will tax these resources away as well when providing $z$, we have

$$\tau w^T = \pi (1 - p) (Z - s^* - \theta).$$

(31)

Substitute for $\tau$ from (31) into the expression for $EU$, we have, upon simplification,

$$EU = wT - \pi (1 - p) (Z - s^*) + [\pi (1 - p) - q] \theta - s^*$$

$$+ (1 - \pi) U (s^*) + \pi [pH (m (\beta)) + (1 - p) H (Z)].$$

(32)

Again, at $\theta = 0$, we have the optimal solution when insurance is being provided solely by the government. To see if having $\theta > 0$ benefits or harms the parents, differentiate (32) with respect to $\theta$. This results in

$$\frac{\partial EU}{\partial \theta} = \pi (1 - p) - q.$$  

(33)

If the private insurance price is fair, we will have $q = \pi$. This implies that $\partial EU/\partial \theta = -\pi p < 0$. Interestingly, then, even fair private insurance markets now cannot do as good a job as public insurance. Intuitively, to the extent that private insurance markets may exist, they do for the possibility of dependency and not the default of altruism. Thus the lowest rate they can offer is $q = \pi$. The opting out scheme, on the other hand, allows the government to effectively provide assistance against the default of altruism and that at an implicit price of $\pi (1 - p)$.20 This is because only the parents who do not receive aid from their children request government's assistance even though all dependent parents can do so.

Clearly, if $q > \pi$, providing assistance through private markets will be even worse.

The main results of this section are summarized in the following proposition.

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20 Fair insurance markets can duplicate this rate only if they can sell insurance against the default of altruism which is not possible.
Proposition 2 Consider an opting out scheme wherein in equilibrium altruistic children help their parents. There exists two types of equilibria: an unrestricted and a restricted.

(i) In both equilibria:

(a) No private insurance is used even if insurance markets are fair.

(b) The optimal amount of LTC insurance, \( g^* = z - s^* \), is always positive and independent of the probability of altruism (as long as \( p < 1 \)).

(ii) The unrestricted equilibrium can occurs only if children are “super altruistic”. In this equilibrium,

(a) Optimal level of insurance is characterized by

\[
H'(s^* + g^*) = 1,
\]

so that there is full LTC insurance.

(b) Comparing the current solution under opting out (OO) with the previous solution under topping up (TU), we have

\[
\begin{align*}
g^{OO} &> g^{TU}, \\
s^{OO} &< s(p; g^{TU}), \\
s^{OO} + g^{OO} &> s(p; g^{TU}) + g^{TU} \\
EU^{OO} &> EU^{TU}.
\end{align*}
\]

(iii) In the restricted equilibrium, there is less than full insurance. Its level, is \( \hat{z} - s^* \) where \( \hat{z} \) is the solution to

\[
H(\hat{z}) = H(m(\beta)) - \frac{m(\beta) - s^*}{\beta}.
\]
5 Optimal opting out policy: altruistic children do not help

The government now maximizes $EU$ subject to 

$$\tau w T = \pi (z - s),$$

as it has to provide $z$ for everyone who is dependent. It thus chooses $z$ to maximize

$$L = w (1 - \tau) T - s^* + (1 - \pi) U (s^*) + \pi H (z),$$

where $s^*$ is the solution to (26). Given the assumption of no help from altruistic children, the solution to the government's optimization problem must satisfy $z > \tilde{z}$ condition.

The first-order condition with respect to $z$ is

$$\frac{dL}{dz} = \pi \left[H' (z) - 1\right].$$

If $H' (\tilde{z}) > 1$, i.e. if $\tilde{z} < z^*$ where $z^*$ is defined by $H' (z^*) = 1$, the solution to this first-order condition is $z = z^*$. The optimal LTC insurance, $g = z - s$, is then characterized by

$$H' (s^* + g^*) = 1,$$

so that we have full insurance. On the other hand, if $\tilde{z} \geq z^*$, increasing $z$ from $\tilde{z}$ does not increase the expected utility of the parents. Under this circumstance, there is no solution wherein altruistic children do not help. (We are assuming that at $z = \tilde{z}$, altruistic children help).

5.1 Private insurance

Consider again the expected utility of the parent, if private insurance markets are operative, and fix the level of $z$ at its optimal level, $z^*$. We have

$$EU = w (1 - \tau) T - s^* - q \theta + (1 - \pi) U (s^*) + \pi H (z^*),$$
where

\[ \tau wT = \pi (z^* - s^* - \theta). \]  

(34)

Substitute for \( \tau \) from (34) into the expression for \( EU \) and simplify to get

\[ EU = wT - \pi (z^* - s^*) + (\pi - q) \theta - s^* + (1 - \pi) U(s^*) + \pi H(z^*). \]  

(35)

Again, at \( \theta = 0 \), we have the optimal solution when insurance is being provided solely by the government. Differentiating (35) with respect to \( \theta \) yields

\[ \frac{\partial EU}{\partial \theta} = \pi - q. \]

Consequently, private insurance will be as good as public insurance if the private insurance price is fair. Otherwise, if \( q > \pi \), public insurance will dominate. Intuitively, when no child helps his parents, public assistance is provided to all dependent parents. This makes its implicit price, when provided publicly, to be \( \pi \) again.

Proposition 3 summarize the results of this section.

**Proposition 3** Consider an opting out scheme. Define \( z^* \) as the solution to \( H'(z^*) = 1 \) and \( \widehat{z} \) as the solution to

\[ H(\widehat{z}) = H(m(\beta)) - \frac{m(\beta) - s^*}{\beta}, \]

where \( s^* \) is the optimal savings of the parents.

(i) If \( z^* \leq \widehat{z} \), i.e., if altruistic children do not consider full LTC insurance to be enough, the optimal policy is to provide full LTC insurance \( (z = z^*) \), and we have the unrestricted equilibrium of Proposition 2.

(ii) If \( z^* > \widehat{z} \), i.e., if altruistic children do not help their parents if they are given full LTC insurance by the government, the optimal policy is either to provide them with full insurance with no assistance from their children; or to set \( z = \widehat{z} \), so that we have the restricted equilibrium of Proposition 2.

(iii) In the equilibrium with no assistance from children, the LTC insurance can be provided privately or publicly if private insurance is fair and publicly otherwise.
6 Concluding remarks

The idea of altruism default is not new. Yet there does not exist much work on it in economics and particularly in the area of long-term care where one expects it to have multiple implications. Even though the family continues to remain the prime source of LTC provision in the US and elsewhere, its role has been on the decline. Among the contributing factors in this trend are mobility of the children, increasing frequency of childless families, fading family norms, and higher labor force participation of middle-aged women. This phenomenon in conjunction with rising levels of life expectancy and an aging population pose challenging questions for policy makers concerning LTC provision. The current paper has studied the role of LTC insurance programs in a world in which family assistance is uncertain. And the main lesson that has emerged from it is that the type of insurance program the government may offer, “topping up” or “opting out,” affects not only the desirability of private versus public insurance but also the desirability of insurance in general and its optimal size (whether private or public).

We have shown that under a topping up scheme, if the probability of altruism is high there is no need for insurance; public or private. At lower probabilities, LTC insurance is called for, albeit one that is less than full. The amount of insurance varies negatively with the probability of altruism. If private insurance markets for dependency are fair, private insurance will suffice (although public assistance is just as good). At higher than fair insurance premiums, public assistance dominates private insurance.

Studying an opting out scheme, the paper has shown the possibility of three equilibria. All require LTC insurance regardless of the probability of altruism (in contrast to the topping up scheme). In one, super altruistic children help their parents even though they are offered full LTC insurance by the government. Under this equilibrium, and in comparison to the equilibrium under the topping up scheme, parents have more LTC insurance and a higher expected utility but less private savings (regardless of the probability of altruism). In the second equilibrium, parents are given full LTC insurance
by the government but are not being helped by their children. In the third equilibrium, the government restricts the LTC insurance to less than full and just low enough to induce altruistic children to help.

The most interesting feature of the opting out scheme is that it provides, indirectly, insurance against not only dependency but also the default of altruism. It does so by creating incentives for self-targeting; ensuring that only dependant parents who are not helped by their children seek help from the government. Consequently, when the equilibrium entails assistance from altruistic children, no private insurance is used even if private insurance markets for dependency are fair. All assistance should be provided publicly. On the other hand, if the equilibrium is one without assistance from altruistic children, public insurance no longer dominates private insurance if the latter is fair. Having touted the advantages of an opting out system, a word of caution is called for. The system works in part because the government has the ability to tax away the dependent elderly’s resources (savings and any private insurance that they may have purchased). This a strong assumption. It is not at all clear that one’s private assets are publicly observable to be taxed away at no cost.

In thinking about fruitful extensions of this work, a number of avenues come to mind. One is to allow for altruism default to be endogenously determined. Parents may be able to influence the likelihood of its occurrence by spending more time with their children instilling family values in them and fostering family solidarity. Another is to introduce heterogeneity in the model. Heterogeneity can take different forms: in parents’ earnings, in the degree of altruism on the part of the children, and in dependency probability. All are realistic and worth studying. One other direction is to probe into the financing the public LTC schemes using a richer tool-box including non-linear income taxes. This is particularly needed in the presence of heterogeneity in earnings. Cremer et al. (2012) study a model wherein altruism default is endogenously determined while simultaneously introducing earnings heterogeneity. However, as in this paper, they assume that the
financing comes from a proportional income tax levied on the parents.

Other research avenues include bringing the welfare of children explicitly into the analysis. In this paper, we have proceeded as if children live in a different community and that we are not concerned about their welfare. An assumption that has made us rule out their taxation as well. If they are in the story, one has to also find a satisfactory way for treating the altruistic component of their utility in the overall social welfare. Simple inclusion gives rise to the possibility of double counting.
References


